From the similar triangles CMB and CAN

$$\frac{a}{m}=\frac{a+y}{\sqrt{(c^2-y^2)}},\text{ or } a^2c^2-a^2y^2=a^2m^2+2am^2y+m^2y^2,$$

whence $$(m^2-c^2)a^2+(m^2+a^2)y^2+2am^2y=0. \qquad (2)$$

Substituting the value of y from (1) we have

$$(m^2-c^2)a^2+(m^2+a^2)\left(\frac{k}{a}-n\right)^2+2am^2\left(\frac{k}{a}-n\right)=0,$$

or $$(m^2-c^2)a^4+(m^2+a^2)(k^2-2akn+a^2n^2)+2a^2m^2(k-an)=0,$$

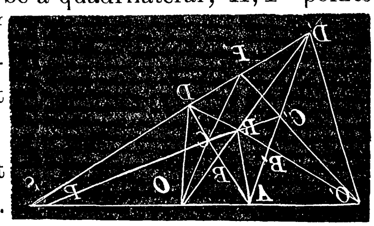
which by expansion and reduction becomes

$$(m^2+n^2-c^2)a^4-2n(k+m^2)a^3+[m^2(n^2+2k)+k^2]a^2-2knm^2a+k^2m^2=0,$$

from which to find a.

NOTE.— This problem has been known in schools under the following form. A tree of known height n, standing on a side hill, was broken over by the wind, and while still clinging to the stump its top touched the ground at a distnce c from the foot of the stump. The perpendicular distance from the foot of the tree to the broken over part was measured and found to be m; required the height of the stump.

DEMONSTRATION OF THE THEOREM OF APOLLONIUS AND ITS RECIPROCAL BY W. E. HEAL.—1. Let $ODD'O'$ be a quadrilateral; A, P' points taken in opposite sides. The diagonals of the quadrilaterals $ODD'O'$, $ODP'A$, $O'D'P'A$ intersect in p'ts that lie in a strai't line, CRC'.



2. In any quadrilateral $AB'RS$ let a p't B be joined to two opposite vertices, A, R. The lines DD', OC, $O'C'$ joining the intersection of opposite sides of the original quadrilateral and of the two derived quadr'ls meet in a point, P'.

To prove 1, produce $D'D$, $O'O$ to meet in P. The lines $ABCD$, $PDP'D'$ cutting the pencil AOD give $[ABCD]^* = [PD'P'D]$. The lines $PDP'D'$ cutting the pencil $AO'D'$ give $[PD'P'D]=[AD'C'B']$; $\therefore$ $[ABCD]=[AD'C'B']$. The points A, B, C, D; A, B', C', D' having the same anhar. ratio and one point, A, common, the lines BD', CC', DB' joining the other corr. points meet in a point R; $\therefore$ C, R, C' are in the same straight line.

To prove 2, join R, A. The lines $ABCD$, $AB'C'D$ cutting the pencil ARD give $[ABCD]=[AD'C'B']$. The pencils $O'OD$, $OO'D'$ having the same anh. ratio and one ray OO' common, the intersections D, P', D' of the other corr. rays lie in a straight line. That is DD', OC, $O'C'$ meet in a point.

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*The notation $[ABCD]$ denotes the anharmonic ratio of A, B, C, D.